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# Thickness Variation in the Thermoforming of Poly(methyl Methacrylate) and High-Impact Polystyrene Sheets

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## Synopsis

The influence of material flow properties on the variation of wall thickness in a thermoformed part was investigated by measuring the thickness reduction at the pole of free-formed axi-symmetric domes of poly(methyl methacrylate) and high-impact polystyrene. It was found that at a given pole height, the thickness reduction in poly(methyl methacrylate) was less than in high-impact polystyrene, i.e., the wall thickness in a part formed from poly(methyl methacrylate) will be more uniform than in a part formed from high-impact polystyrene by the same technique. This difference in formability was ascribed to a difference in the dependence of the flow stress $\sigma$ at the thermoforming temperatures on time. The flow stress of both materials was given by $\sigma = Kt^{m'}\epsilon^n$, but whereas $n$ was approximately 1 for both materials, $m'$ was $-0.052$ and $-0.33$ for poly(methyl methacrylate) and high-impact polystyrene, respectively. A physical argument and simple analysis led to the conclusion that a large (negative) value of the "stress relaxation index" in a material reduces the degree of uniformity of sheet thickness in a formed part.

## INTRODUCTION

The wall thickness of a thermoformed part generally has to lie between tolerances set by the designer. Techniques, such as air-slip forming, plug-assist, and billow snap-back can be employed to achieve these tolerances;[1] however, whatever technique is employed, the thickness is never uniform and the variation in thickness in combination with the tolerances on thickness must set a limit on the possible depth-to-width ratio of a thermoformed part. Factors other than technique that affect the variation in thickness are, therefore, of some interest. Temperature and its variation across the sheet during thermoforming are such factors, but an important factor, if temperature variation is negligible, is the flow property of the plastic: plastics with different flow properties should result in parts with different degrees of thickness variation when formed with identical techniques.

The research described in this paper had as its aim the exploring of the relation between the variation in thickness and the flow properties by measuring the variation in thickness resulting from a particularly simple forming operation, the free forming of an axi-symmetric dome from a circular blank, and comparing the results with stress–strain–time data obtained from tensile testing of the plastics at the forming temperatures. Two plastics were used: poly(methyl methacrylate) (PMMA) and high-impact polystyrene (HIP).

Previous related work has been by Alfrey[2] and Williams.[3] Alfrey discussed in a general way procedures for analyzing thermoforming. Williams analyzed the bulging of PMMA to various shapes. The solution he gives for the thickness variation over an axi-symmetric free-formed dome is the same as derived by Hill.[4] As Hill points out, the solution is for a material with a stress–strain behavior given by $\sigma = \sigma_0 \exp \epsilon$, where $\sigma$ is true stress, $\sigma_0$ is a constant, and $\epsilon$ is the true strain. PMMA at thermoforming temperatures does not follow this stress–strain behavior, so the solution given by Williams cannot be applicable.

The main interest of the present work was axi-symmetric free forming; however, two-dimensional (plane-strain) free forming was also examined. This was achieved by bulging a sheet through a rectangular die with a length-to-width ratio of 2.9:1.

## EXPERIMENTAL

### Apparatus

Figure 1 shows the forming apparatus: an infrared heater and a pneumatically actuated forming press approximately 3 feet away from the heater. Hot plastic blanks are carried from the heater to the press on the trolley shown.

The heater is not enclosed or insulated. It consists of two banks of six Elstein 650-watt, glazed ceramic infrared trough elements, the power to which is controlled by variacs. The gap between the banks is adjustable. The plastic blanks are carried by the trolley into the gap for heating.

The forming press consists of a pneumatic cylinder which raises and lowers a clamping plate, and a steel box into which air pressure is applied for forming. The steel box is fitted with an O-ring which prevents air escaping from under the

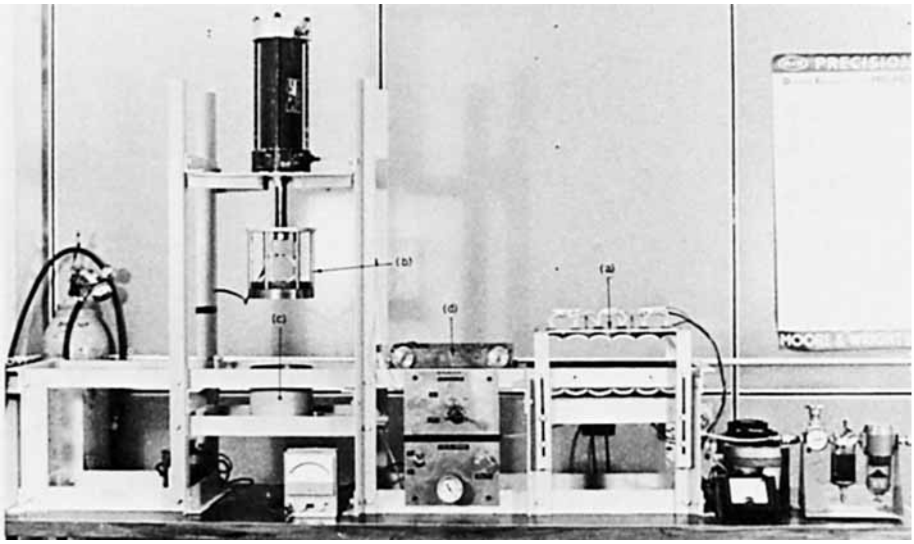

Fig. 1. The thermoforming apparatus: (a) heater; (b) clamping plate; (c) steel box into which compressed air is fed; (d) trolley.

TABLE I
Values of $m'$ and $n$ in $\sigma = Kt^{m'}\epsilon^n$

| Material | $m'$ | $n$ |
|---|---|---|
| PMMA | −0.05 | 1.0 |
| HIP | −0.33 | 1.1 |

TABLE II
Dimensions of Blanks

| Case | Material | Thickness, in. | Dimensions, in. |
|---|---|---|---|
| Axi-symmetric | PMMA | 0.246 ± 0.004 | 7 (diam.) |
|  | HIP | 0.127 ± 0.005 | 7 (diam.) |
| Two-dimensional | PMMA | 0.104 ± 0.002 | $11^1/_2 \times 4$ |
|  | HIP | 0.127 ± 0.005 | $11^1/_2 \times 4$ |

edge of the clamped plastic blank during forming. The air for forming is not preheated.

## Materials

Two materials were investigated: $^1/_8$-in. and $^1/_4$-in. clear poly(methyl methacrylate) (PMMA) sheet supplied by Imperial Chemical Industries Ltd, and $^1/_8$-in. white opaque high-impact polystyrene (HIP) sheet supplied by Garnite Plastics Ltd. From tensile tests at constant cross-head speed on specimens machined from the sheets, it was established that at the temperatures mainly used in the thermoforming investigations (163° and 122°C for PMMA and HIP, respectively), the (true) flow stress $\sigma$ of the plastics was given by

$$\sigma = Kt^{m'}\epsilon^n \quad (1)$$

where $K$ is a constant, $m'$ is an index of stress relaxation, and $n$ is a strain hardending index.[5] Values of $m'$ and $n$ from reference 5 are given in Table I. Both materials are rubber-like and viscoelastic at the forming temperatures.

The dimensions of the blanks for thermoforming are given in Table II.

## Procedure

It was necessary, first of all, to ascertain the combination of power to the heater and heating time to bring the plastic blanks to the desired temperatures. For one power setting, the trolley without any plastic blank was heated in the heater for 45 to 60 min. A plastic strip, 1 in. in width and with a length equal to either the diameter of the blank in axi-symmetric forming or the width of the blank in two-dimensional forming, machined from the same sheets as in thermoforming, was then placed on the trolley and heated between the heating elements. Temperatures of the strip at the position of the center of a thermoforming blank and at various distances from the center were measured at various times using thermocouples embedded in the plastic at midthickness. This procedure was repeated at various power settings. It was found that a low power setting and a large heating time was necessary to minimize temperature differences between the center of the blank and the edges. A heating time of 25 min was finally selected. Most of the thermoforming investigations were at a power setting which after 25 min of heating time gave blank temperatures of 165° and 125°C

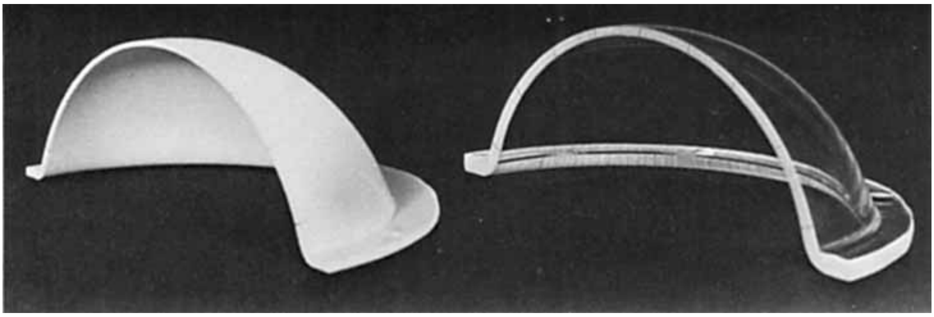

Fig. 2. Axi-symmetric domes of PMMA and HIP, sectioned for thickness measurement.

for PMMA and HIP, respectively. At this power setting the temperature over the blanks at any time during heating varied by less than 4°C. There was no discernible pattern to this variation: sometimes the edges were slightly hotter than the center and vice versa.

To carry out the actual thermoforming, first the trolley was heated for 45–60 min. A plastic blank was then placed on the trolley and heated for 25 min after which it was quickly transferred on the trolley to the forming press. The plastic sagged on heating, HIP sagging the most. The amount of sag could be as much as $^1/_2$ in. Once in the forming press, the edges of the blank were clamped and the blank formed with a positive air pressure. It took approximately 10 sec to transfer a blank from the furnace and completely form it. The actual forming operation averaged 6 sec. From cooling curves obtained with the test strips, it was established that the temperatures of the materials during forming for most of the investigations were 163° and 122°C for PMMA and HIP, respectively. Forming was also done at other temperatures: 150° to 170°C for PMMA and 110° to 130°C for HIP. The formed parts were allowed to cool before removing them from the press.

The formed parts were sectioned for thickness measurement. In the axi-symmetric domes, sectioning was along a meridian (Fig. 2). The thickness was measured at various positions along the meridian using a micrometer. In the two-dimensional case, strips spanning the width of the bulge were cut from the central portion of the length of the bulge and thickness was measured at various positions on these strips with a micrometer.

Pole heights and deflections at other points were measured by projecting the profile of the sections onto paper (Figs. 3 and 4). The radii of curvature of the formings could be measured by fitting circles of known radius to the projected profiles.

In the two-dimensional forming, grids of $^1/_2$ cm square were marked on the blanks prior to forming to investigate the amount of longitudinal strain in the forming and to ensure that two-dimensional (plane-strain) conditions prevailed over at least a central portion of the length of the bulge.

## RESULTS AND DISCUSSION

### Axi-Symmetric Domes

It was found that over the range of temperatures investigated (150–170°C and 110–130°C for PMMA and HIP, respectively) the forming temperature had no

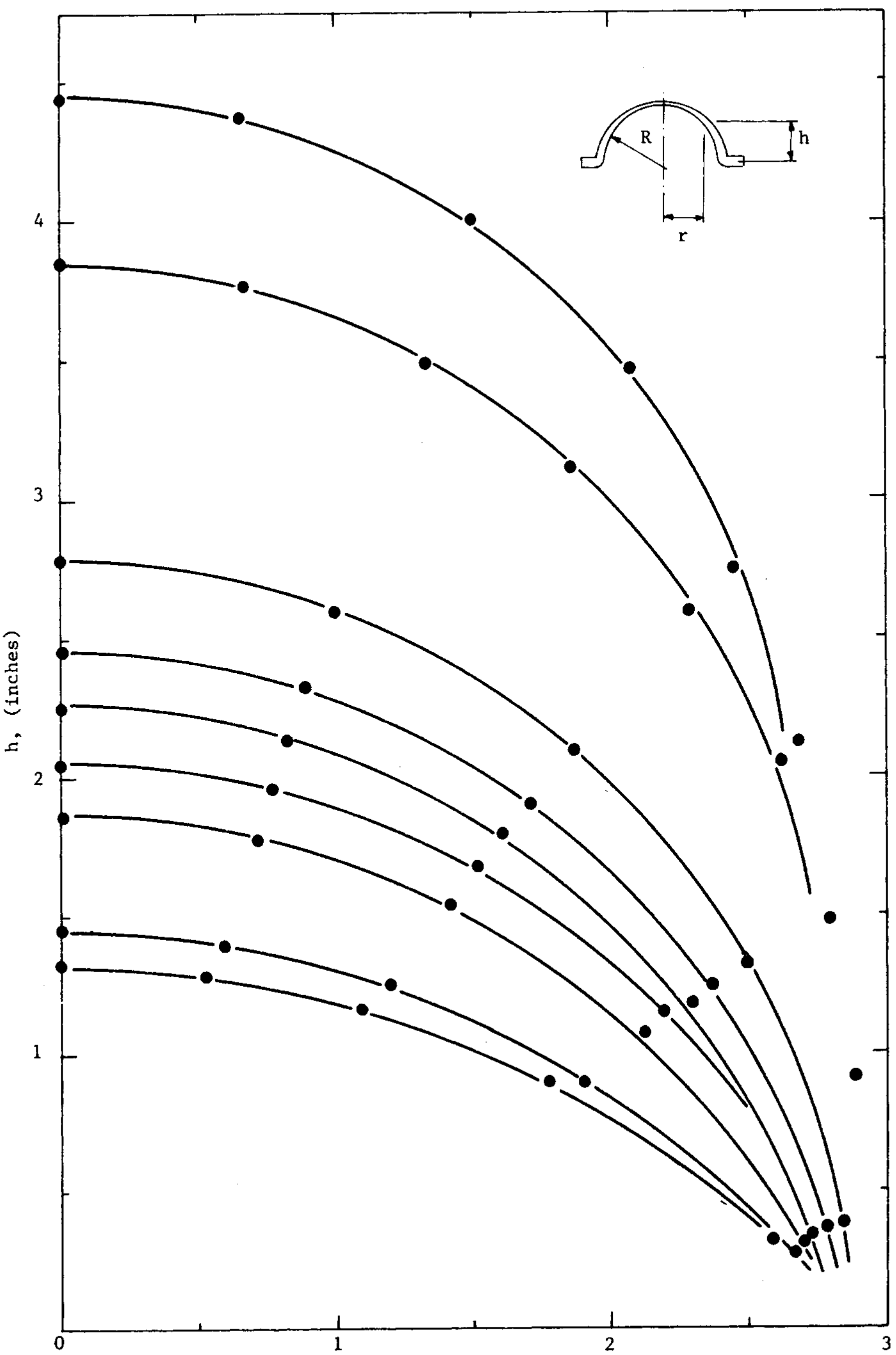

Fig. 3. Profiles of axi-symmetric domes of PMMA, thermoformed at 163°C. The curves shown have a constant radius of curvature *R*.

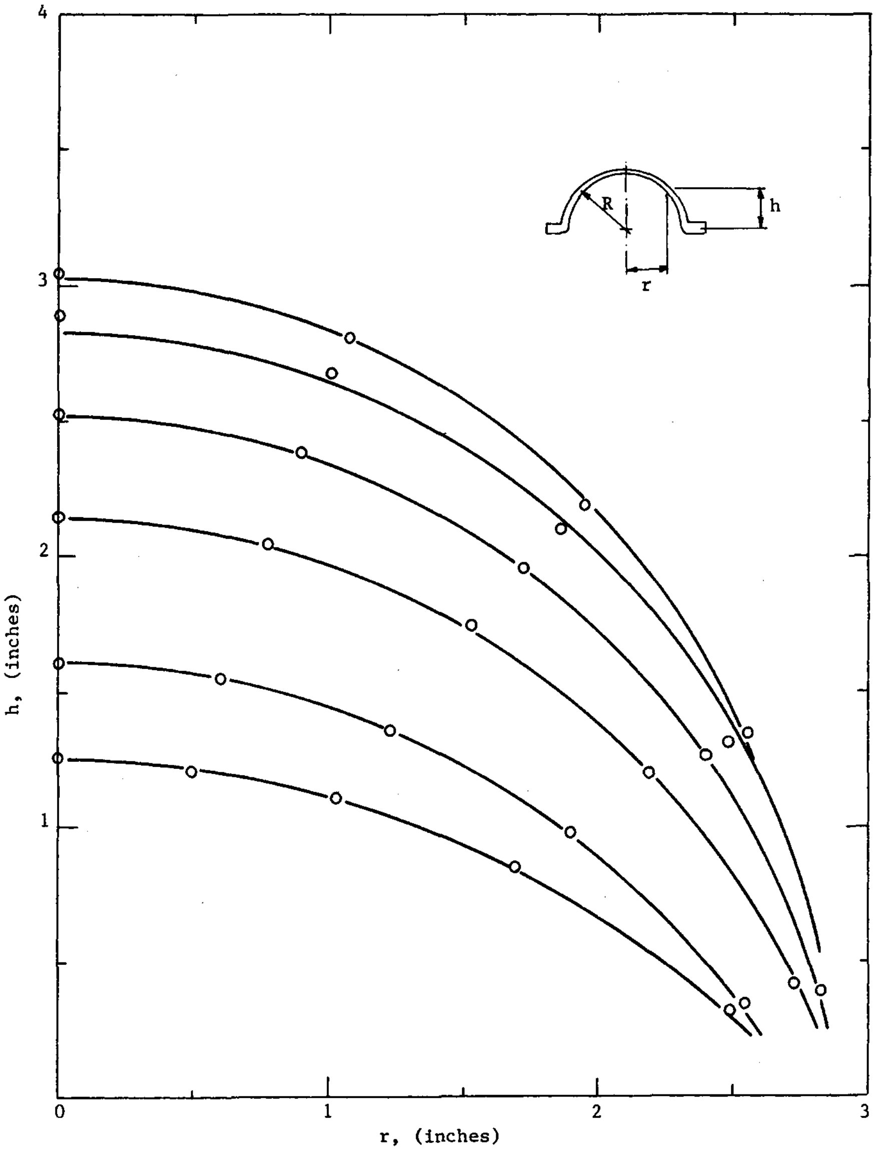

Fig. 4. Profiles of axi-symmetric domes of HIP, thermoformed at 122°C. The curves shown have a constant radius of curvature *R*.

effect on the thickness distribution and the shape of the domes. The results that follow, however, are for forming at 163° and 122°C for PMMA and HIP, respectively.

Figure 5 gives the thickness reduction at the pole, $1 - (S_p/S_0)$, where $S_p$ is the thickness of the pole and $S_0$ is the initial sheet thickness as a function of the ratio of pole height $H$ to dome base radius $a$, $H/a$. For a given value of $H/a$, the thickness reduction at the pole is less in PMMA than in HIP, i.e., PMMA shows superior formability in this respect. The ratio $H/a$, though conveniently measured, is not a significant physical parameter. A better representation of the average deformation undergone in the forming is the mean thick-

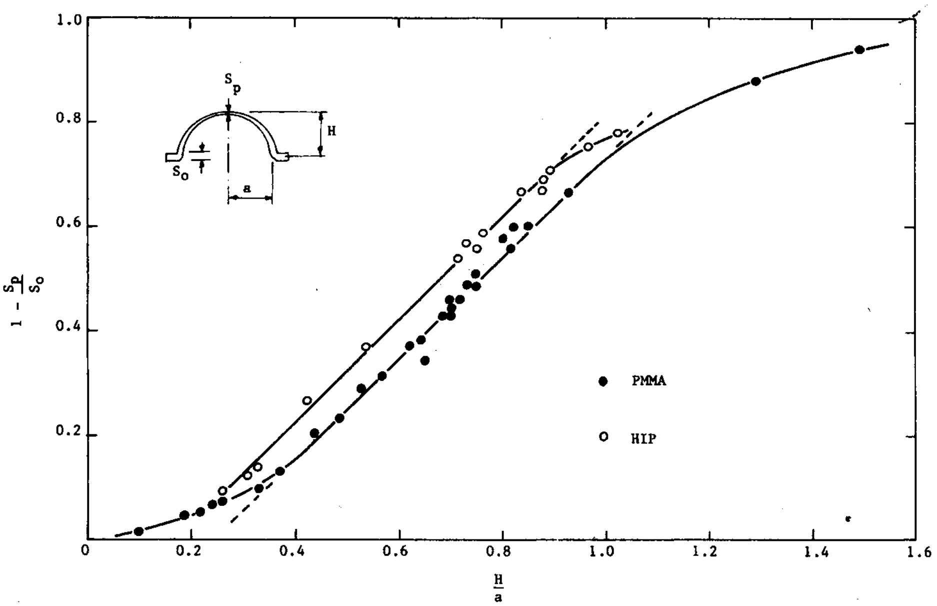

Fig. 5. Variation of thickness reduction of the pole 1 — $(S_p/S_0)$, with pole height $H$ and base radius $a$ for axi-symmetric domes of PMMA and HIP.

ness reduction, 1 — $(\bar{S}/S_0)$. This is obtained by measuring the length of a meridian on the bulge, C(6): by constancy of volume of plastic

$$\frac{\bar{S}}{S_0} = \left(\frac{2a}{C}\right)^2.$$

Figure 6 is a plot of the data of Figure 5 in the form 1 — $(S_p/S_0)$ versus 1 — $(\bar{S}/S_0)$. In this representation the tails of both ends of the range in Figure 5 are absent. It can be seen that for a given mean thickness reduction HIP always has a larger deviation from uniform thickness than PMMA.

This difference must have a basis in the strain and time dependence of the flow stress of the materials. If we consider two positions on a dome, the thickness at the first position being less than at the second, a number of effects will influence the magnification of the thickness difference with further increases in pressure. That the stress in the sheet is greater at the thinner position acts to magnify the thickness difference. However, the material at the thinner position has undergone more strain and the flow stress there will be greater than at the thicker position—an effect counteracting a magnification of the thickness difference. The increase in the flow stress with strain depends on the inherent rate of strain hardening of the material and also on the rate of relaxation of the flow stress with time. We would expect, therefore, that a high rate of strain hardening and a low rate of stress relaxation will act to increase the thickness uniformity. A simple analysis (see Appendix) shows that the incremental increase in a thickness strain difference between two neighboring points on a sheet bulged to a spherical cap of radius $R$ is given by

$$\frac{\delta \Delta \epsilon}{\Delta \epsilon} = \frac{1}{n - \epsilon}\left(\frac{\delta p}{p} + \frac{\delta R}{R} - m'\frac{\delta t}{t}\right) \tag{2}$$

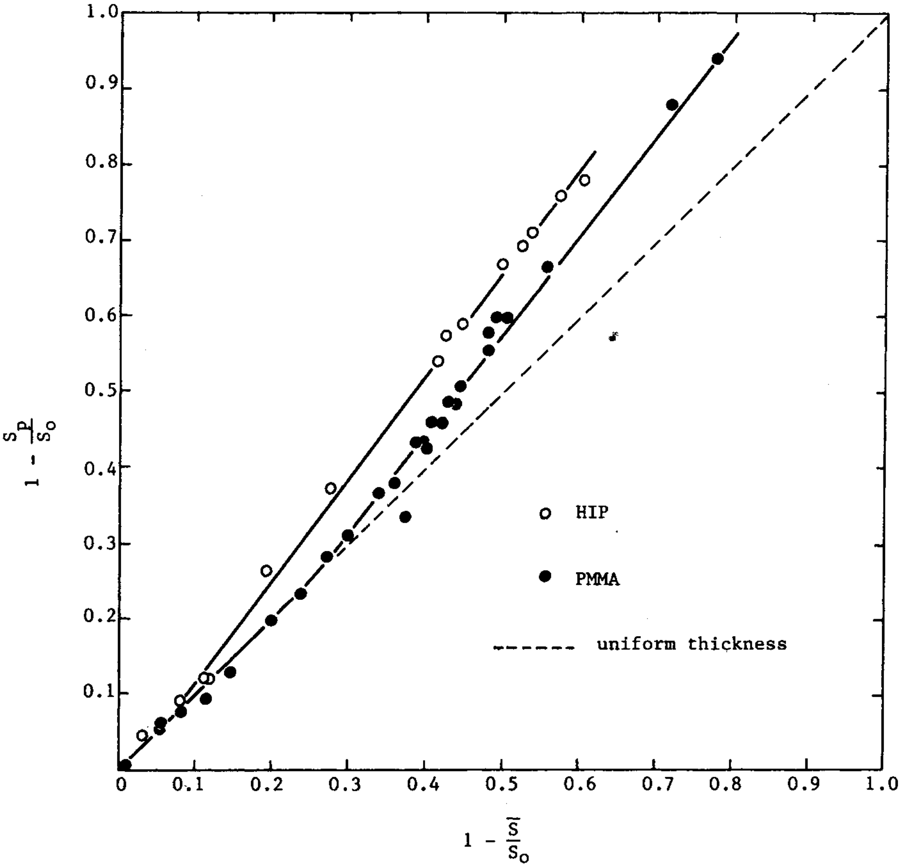

Fig. 6. Variation of thickness reduction at the pole $1 - (S_p/S_0)$ with mean thickness reduction $1 - (\bar{S}/S_0)$.

where $\Delta\epsilon$ is the difference in strain between two neighboring points the mean of whose strain is $\epsilon$, $p$ is the pressure, and $t$ is the time. It will be remembered that $n$ and $m'$ are indices of strain hardening and stress relaxation, respectively, in the equation for the flow stress:

$$\sigma = Kt^{m'}\epsilon^n. \tag{1}$$

The conclusion from eq. (2) is that a high index of strain hardening and a small index of stress relaxation favors thickness uniformity. The strain hardening indices of PMMA and HIP are approximately identical (Table I); however, $m'$ is $-0.33$ and $-0.052$ for HIP and PMMA, respectively. It is suggested that the difference in the indices of stress relaxation causes the difference in thickness uniformity shown in Figure 6.

## Two-Dimensional (Plane Strain) Free Forming

Examination of the distortion of the $^1/_2$-cm square grids on the formed plastic sheets showed that longitudinal elongation within 4 in. of the midlength of the dome was less than 3%, i.e., plane-strain deformation occurred in that region.

Transverse profiles were circular except near the clamped edges. Thickness distribution is shown in Figures 7 and 8. Apart from some unexplained devia-

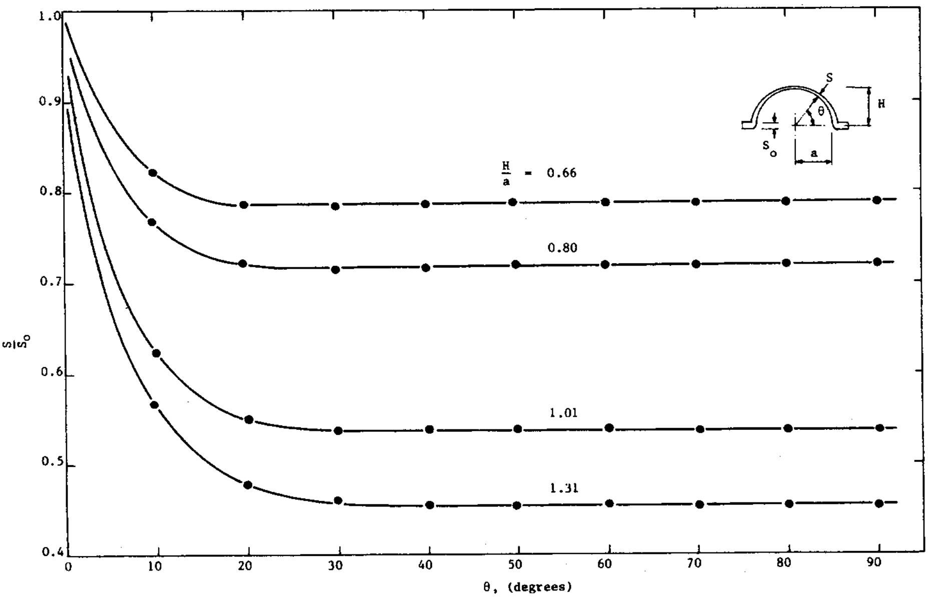

Fig. 7. Thickness distribution in two-dimensional thermoforming of PMMA at 163°C.

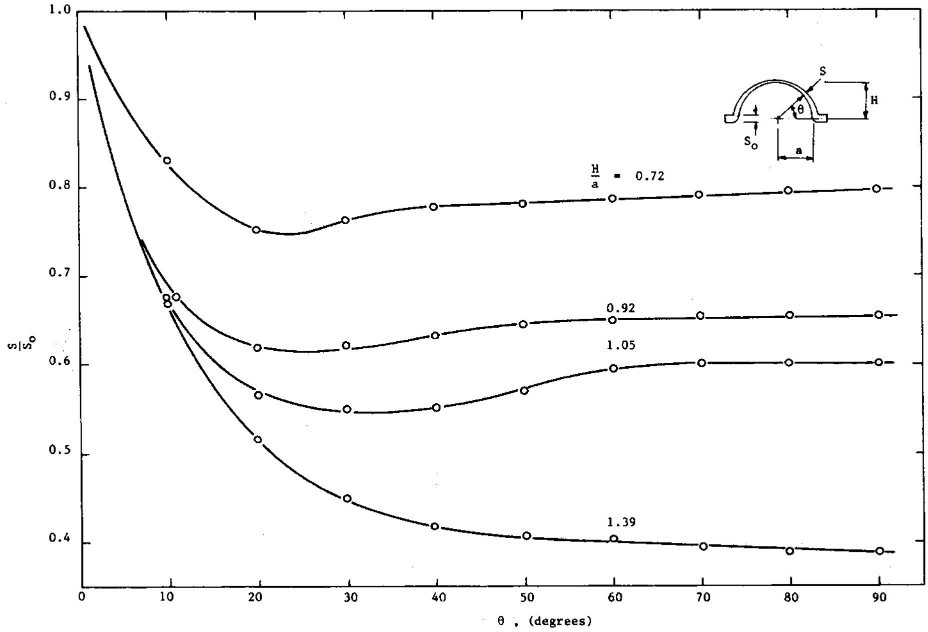

Fig. 8. Thickness distribution in two-dimensional thermoforming of HIP at 122°C.

tion for HIP, the trend is for the thickness to be uniform except near the clamped edges.

On a little reflection, it is clear that two-dimensional plane-strain free forming of any material should result in a uniformly thick circular section, at least away from the clamped edges. The present results tend to confirm this. It was found that the constancy of volume assumption predicted the thickness in the constant-thickness portion of the profiles to an accuracy of 2% to 5%.

## CONCLUSIONS

The high degree of uniformity in thickness that is found in forming plastics is due to the large-strain hardening capacity of plastics at thermoforming temperatures. Stress relaxation (viscoelasticity) reduces the degree of uniformity in thickness.

## Appendix

### Analysis

Consider a plastic sheet, initially flat, clamped by a circular ring at its edges and expanded through the ring by a pressure $p$ to form at time $t$ a spherical cap of radius $R$; $S$ is the sheet thickness at a particular point on the cap. For equilibrium,

$$\sigma = \frac{pR}{S} \tag{1A}$$

where $\sigma$ is the balanced biaxial stress in the cap.

The stress is also given by

$$\sigma = Kt^{m'}\epsilon^{n} \tag{2A}$$

which is eq. (1) of the main text.

Eliminating $\sigma$ yields

$$n\frac{\delta\epsilon}{\epsilon} = \frac{\delta p}{p} + \frac{\delta R}{R} - \frac{\delta S}{S} - m'\frac{\delta t}{t} \tag{3A}$$

where $\delta\epsilon$, $\delta p$, $\delta R$, and $\delta S$ are incremental changes during an incremental time $\delta t$. Now,

$$\frac{\delta S}{S} = -\delta\epsilon \text{ (balanced biaxial strain).}$$

Therefore,

$$\delta\epsilon = \frac{\epsilon}{n-\epsilon}\left(\frac{\delta p}{p} + \frac{\delta R}{R} - m'\frac{\delta t}{t}\right). \tag{4A}$$

The difference in strain increment between two neighboring points on the cap, strained to $\epsilon$ and $\epsilon + \Delta\epsilon$, is given by

$$\frac{\Delta\delta\epsilon}{\Delta\epsilon} = \frac{1}{n-\epsilon}\left(\frac{\delta p}{p} + \frac{\delta R}{R} - m'\frac{\delta t}{t}\right). \tag{5A}$$

Thus, enhancement of the thickness (strain) difference between the two points is discouraged by a high value of $n$ and a small value of $|m'|$ (note that $m'$ is a negative index).

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